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Quantum Logical Interpretation of Quantum Mechanics: The Role of Time

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1. INTRODUCTION

Discovery of Bell's (1964) inequalities and experimental evidence of their breaking (Aspect *et al.*, 1982) gave rise to an increase of interest of the scientific community in the problem of the interpretation of quantum mechanics. If one takes the formalism of quantum theory and all the experimental support for it, one must say that the so-called theories of hidden variables seem very unprobable today if one does not want to say goodbye to the theory of relativity, which is too high a price for philosophical prejudice. So the Copenhagen interpretation with its stress on the role of the observer, indeterminism, "objectively existing potentialities," and complementarity is now the only interpretation consistent with the textbooks on quantum mechanics.

Any other interpretation is not on the level to be taught to students because it either contradicts known experimental facts or is very undeveloped. First it leads to some unobserved experimental differences with usual quantum mechanics, and second all experimental data from elementary particle physics to chemistry are not described by such a theory, but are described by usual quantum theory. But what is the Copenhagen interpretation? If one tries to summarize all that was said by N. Bohr, W. Heisenberg, J. von Neumann, and V. A. Fock, one will see that one still can have different "interpretations of the Copenhagen interpretation."

One of these interpretations—the positivistic—can mean that quantum "objects" do not exist at all, only "classical" objects exists, and what

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we call "quantum objects" is just the language for the description of the behavior of classical preparing and measuring apparatuses in some unusual classical situations.

The word "*classical*" here means that all these apparatuses can be described by usual classical physics and usual language. Some words of N. Bohr can lead to this interpretation, with its claim on the "unreality of quantum objects."

The other "interpretation of the Copenhagen interpretation" can be called "realistic": Quantum objects are real, but this reality is not the same as the reality of classical objects. As W. Heisenberg and V. A. Fock put it: quantum reality corresponds to "objectively existing potentialities."

V. A. Fock was the teacher of the present author in quantum mechanics, so it is this interpretation which will be discussed here.

"Potentialities" usually exist not "objectively," but "subjectively." They are ideas in our mind about reality. For example, if somebody shoots a cannon, the explosion can potentially be in many different places if we do not "know" the trajectory. Before observation of the explosion, the observer can draw many different trajectories of the explosive—they are potentialities existing only in his mind. When he observes the event, "potential" becomes "actual" and he "discovers" one objectively existing trajectory. In contrast, in quantum mechanics one has "objectively existing" potentialities which are totally different from anything we have in usual classical physics.

One of the attempts to give a "realistic" interpretation of quantum mechanics consistent with its formalism is the Everett–Wheeler–DeWitt many-world interpretation. But we think that this interpretation, which uses such words as "worlds," "splitting of the world," "many copies of the same observer" in different worlds without giving the exact meaning of these words is a bad language for "objectively existing potentialities." For example, it is impossible to use the word "worlds" in the sense of "world of events" in some space-time. It is impossible to use the word "splitting" if one really thinks that "one" electron with definite spin projection S_z becomes "two electrons" when one measures S_x, because this contradicts the conservation of charge. The same with splitting of the observer, which also contradicts conservation laws. In spite of the impossibility of observation of this nonconservation, the "metatheory" must be the theory which has these features. If one tries to struggle with this problem by postulating from the beginning of the existence of an infinite number of identical particles ("universes") which become nonidentical due to measurement, one still has problems with the description of the process of "splitting" in terms of quantum field theory. Why in measurement must one choose one preferable basis but not others? In the usual Hilbert space description there is no preferable basis: this notion is foreign to usual quantum mechanics and so as in hidden-variables theory this leads to a departure from usual quantum mechanics, contrary to hopes of the authors of the EWW interpretation. So in this paper we will discuss other "realistic" interpretations of quantum mechanics, which we shall call the "quantum logical" interpretation. Developed by Birkhoff and von Neumann (1936), Piron (1976), and Finkelstein (1963), it can give a more consistent language for a "realistic" version of the Copenhagen interpretation. We discuss Bell's inequalities and their breaking in the quantum logical approach, the role of the observer, and the specific role of time.

2. BELL'S INEQUALITIES, QUANTUM LOGIC

As is well known, Einstein, Podolsky, and Rosen (1935) in their famous paper claimed that they believe that in spite of the impossibility to measure simultaneously complementary observables for quantum objects such as coordinates and momenta, spin projections on different axes, etc., properties described by noncommuting operators in the formalism of quantum theory "exist" as elements of reality. This must mean the incompleteness of quantum theory because, if description by the wave function is complete, then there is no common eigenfunction for noncommuting operators and so there cannot be a state for which both observables have definite values.

A serious blow to the EPR idea of "elements of reality" was made by Bell's (1964) theorem. Let us take three observables A, B, C which take ± 1 values to which there correspond three noncommuting operators $\hat{A}, \hat{B}, \hat{C}$ (for example, three spin projections $\hat{S}_x, \hat{S}_y, \hat{S}_z$ for spin 1/2). Suppose with EPR that properties A, B, C "*exist*" as elements of reality. Then if $N(A, B, C)$ is the number of particles with definite A, B, C, one comes to simple equalities:

$$\begin{aligned} N(A^+B^-) &= N(A^+B^-C^+) + N(A^+B^-C^-) \\ N(B^-C^+) &= N(A^+B^-C^+) + N(A^-B^-C^+) \\ N(A^+C^-) &= N(A^+B^+C^-) + N(A^+B^-C^-) \end{aligned} \tag{1}$$

from which one comes to

$$N(A^+B^-) \le N(B^-C^+) + N(A^+C^-) \tag{2}$$

This is one form of Bell's inequalities. In quantum theory they can be broken and experiment shows that sometimes they are broken just in those cases when it is predicted by quantum theory.

The meaning of breaking the inequalities (2) can be twofold.

1. Properties of quantum objects described by noncommuting observables and "quantum objects" themselves do not exist as elements of reality if they are not observed. If we observe spin projection S_x, we "create" it by our observation, in this experiment we "prepare" the wave function u_k which is the eigenfunction of $\hat{S}_x$. If next we observe S_y, we "create" a new reality corresponding to $\hat{S}_y$, the wave function will be changed to v_k—the eigenfunction of $\hat{S}_y$.

According to the projection postulate of von Neumann, the wave function during measurement "jumps" discontinuously from u_k to v_k and the probability of a definite result v_k is calculated according to the rule: Write

$$u_n = \sum_k c_{nk} v_k$$

Then $|c_{nk}|^2 = p_{nk}$ is the probability of transforming u_n to v_k.

In quantum field theory "particle number operator" $\hat{N}$ does not commute with local operators, for example, current density $\mathbf{j}(\mathbf{x}, t)$. That is why "one electron" exists as "one" only because an observer "made a choice" to measure $\hat{N}$ but not complementary to it $\mathbf{j}(\mathbf{x}, t)$. To measure a local observable for an electron–positron field one must do measurements for distances smaller than the Compton length of the electron. But what does it mean: properties of quantum objects and quantum objects themselves exist only when they are observed?

A simple answer can be: there are no quantum objects as real objects, only classical apparatuses exist; "quantum objects" is the name for correlations between the behavior of these apparatuses in special situations. For example, electrons, protons, etc., are correlations between certain manipulations with accelerators and bubble chambers etc. The same is true for atoms, molecules, etc. (Lüdwig, 1985). This "nonexistence of quantum objects" in some sense is similar to the belief of a superstitious man when he claims that there is a correlation between the appearance of a black cat crossing his way and an unpleasant event in the end of this way. Surely this man will say: there is no "carrier" of energy from the black cat to the bad event, there is only a "correlation." If he'll see this correlation many times he can name this correlation by some special term "caton"!

So in some cases correlations between classical events are described by a "carrier" (if the black cat itself does something bad: bites the observer, etc.) and this is classical physics. Sometimes there is no "carrier"—this is quantum physics. Von Neumann's "jump" of the wave function is not some "objective process"—there is no objective jump like an objective collapse of the wave function; simply the correlation is changed if macroscopic apparatuses are changed (for example, if one prepares definite S_x and then measures S_y or S_z one will use different rules to find "correlations"). The objection to this "nonexistence theory" is evident: classical macrobodies consists of quantum objects and many of their properties can be obtained (for instance, for low temperatures) from quantum theory, but not the opposite! So one comes to the second possibility:

2. Quantum objects and their properties "exist" but this existence is some new kind of existence. They exist as "objective potentialities." Bell's inequalities are valid if one has a distributive law for A, B, C, for example,

$$A^+B^- \wedge (C^+ \vee C^-) = (A^+B^- \wedge C^+) \vee (A^+B^- \wedge C^-) \tag{3}$$

which is behind the first formula in (1).

But if one thinks together with Birkhoff and von Neumann (1936) that the distributive law is incorrect for quantum objects, one can have

$$A^+ \wedge (C^+ \vee C^-) \neq (A^+ \wedge C^+) \vee (A^+ \wedge C^-) \tag{4}$$

So one comes to the "quantum logical" interpretation of quantum mechanics. According to it, in the double-slit experiment one can say that an electron with definite momentum passes through this "or" that hole, which is not the same as "either" this "or" that hole.

Conjunction $\wedge$ and disjunction $\vee$ in quantum logic do not satisfy the distributive law. A "quantum object" is the "nondistributive lattice" of its properties. Nondistributivity corresponds to noncommutativity of observables.

This nondistributive lattice "exists" as a structure objectively and this can be called a "quantum object." Nevertheless, quantum properties do not correspond to "events" in space-time. This is easy to see from the following example. Let us discuss the EPR experiment for two spin-1/2 particles prepared in the singlet state.

One can ask the following question (d'Espagnat, 1976): if somebody measures spin projection $S_z = +1$ for particle 1, then one can say that with probability 1, spin projection for particle 2 will be -1. But does this $S_z^{(2)} = -1$ correspond to an "event" in space-time if nobody observed $S_z^{(2)} = -1$ in point 2? It is easy to see that this cannot be an event in spite of probability equal to one. If 2 is an event, then one can have a reference frame where simultaneously to 1 one has not 2, but 2′ (before 2) and there is also a reference frame where simultaneously with 2 one has 1′ previous to 1, etc. (Fig. 1). But then it will mean that before observation both $S_z^{(1)} = 1/2$ and $S_z^{(2)} = -1/2$ preexisted as elements of reality with their "truth values" as "true." But then one comes to Bell's inequalities because S_z is not better than S_x, S_y.

The structure of Minkowski space-time as an "event" space-time corresponds to usual set theory, so that one has the distributivity property.

So in the quantum logical interpretation, truth values for properties must be given by a Boolean observer. Without the observer "true" is only a nondistributive "structure" of the lattice.

So, as in any version of the Copenhagen interpretation, the observer plays an important role in the quantum logical interpretation. He gives truth values to elements of a nondistributive lattice according to his Boolean logic. Non-Boolean, nondistributive logic is nonhuman logic; the observer must "translate" nonhuman quantum logic into his Boolean language. This "nonhumanness" of quantum logic is the expression of the "objectivity" of quantum objects. They are not human ideas, just because they cannot be described by human Boolean logic. Nevertheless they are not "objective events": they correspond to some "veiled reality" (d'Espagnat's term). Reality we see as observers is the result of the "interaction" of the observer and objective nonhuman nondistributive reality.

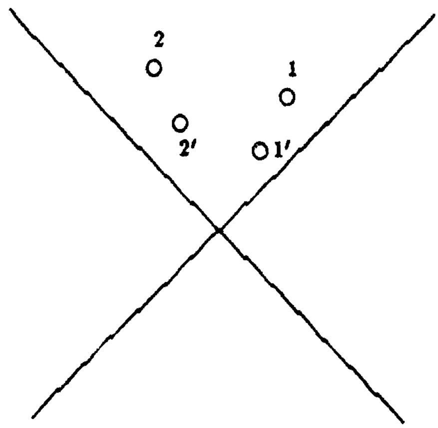

Fig. 1

3. THE ROLE OF OBSERVER IN QUANTUM LOGIC

Here we give a simple example of a nondistributive lattice of the particle with spin 1, spin projections $\hat{S}_z$, $\hat{S}_y$ of which can be measured. We illustrate the main ideas of what we call the "quantum logical interpretation."

Let us draw the Hasse diagram for this particle (Fig. 2).

Here 1, 2, 3 correspond to YES–NO questions: is $S_z = 1$? is $S_z = 0$? is $S_z = -1$? Elements 4, 5, 6 correspond to $S_y = 1, 0, -1$. Properties 1–6 correspond to logical "atoms": they are exclusive in the sense: $1 \wedge 2 = 1 \wedge 3 = 1 \wedge 4 = \cdots = 3 \wedge 6 = \cdots = 5 \wedge 6 = \emptyset$ where $\emptyset$ means "false." Two lines going "up" intersect in "or," which means disjunction, two lines going "down" intersect in "and." So $9 = 1 \vee 2$, $7 = 2 \vee 3$, $8 = 1 \vee 3$, $10 = 5 \vee 6$, etc.

Element I means "always true." If lines are "wires," elements are "lamps," and one can look at the Hasse diagram in terms of an electric current going from $\emptyset$ to I, so that if 1 is "bright," then 8 is "bright" and I is "bright"; 2–6 then are "dark."

The lattice corresponding to the Hasse diagram of Fig. 2 is a complemented orthomodular lattice. To any element corresponds its complement: to $1 \to 7$, $7 = 1'$, and orthogonality is defined: $a^{\perp} = b < a'$, where $<$ means partial order in the lattice ($a < b$ means a true $\to b$ true).

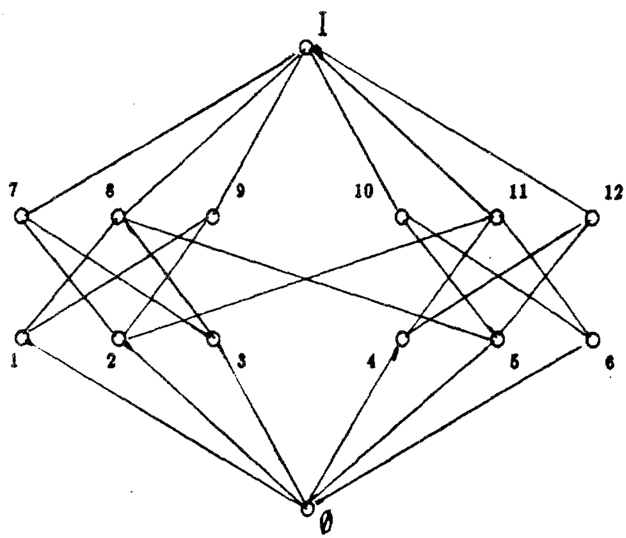

Fig. 2

To 1 the orthogonal elements are 2, 3; to 2 these elements are 1, 3; etc. The main features of our lattice are:

(a) It is nondistributive

$$2 \wedge (4 \vee 5 \vee 6) = 2 \wedge I = 2 \neq (2 \wedge 4) \vee (2 \wedge 5) \vee (2 \wedge 6)$$

(b) Contrary to usual Boolean logic, it can be that even if a, b, c are "false," $a \vee b \vee c$ is "true." One can have: 1 is "true," 4, 5, 6 are "false," but $4 \vee 6 \vee 6 = I$ is "true." This unusual property of disjunction means that the disjunction $a \vee b$ is "true" when a "true" or b "true" and not "only when."

Our Hasse diagram consists of two distributive parts,

$$1 \wedge (2 \vee 3) = 1 \wedge 7 = \emptyset = (1 \wedge 2) \vee (1 \wedge 3)$$

$$(4 \wedge (5 \vee 6) = 4 \wedge 10 = \emptyset, \quad \text{etc.}$$

As to the properties of conjunction, our "electrical wire" analogy is not good. It is possible to have 1 "true" but also 4 "true," 5 "false," 6 "false," and $4 \wedge 5 \wedge 6$ "false," $1 \wedge 4 \wedge 5 \wedge 6$ "false." This means that "truth values" can be given to complementary observables arbitrarily. One can have 1 "true," 2, 3 (orthogonal) "false," but in spite of $1 \wedge 4 = 1 \wedge 5 = 1 \wedge 6 = \emptyset$, one can have 4 "true." Instead of 4 one can take 5 or 6. It is impossible to have 4 "true," 5 "true," 6 "true," because 5, 6 are orthogonal to 4. Any atom of this nondistributive lattice is in the same ambiguous position. That is why one can call these properties "objectively existing potentialities." And here the observer plays an important role. Our postulate will be: *The "observer gives truth values to elements of the lattice according to his (her) Boolean logic.*" This corresponds to the von Neumann projection postulate.

One comes to the following picture. A Boolean-minded observer contemplates a non-Boolean lattice: let he (she) says 1 is "true." Then, being Boolean-minded, he (she) says 2–6 are all "false." But the "objectively existing" nondistributive structure of the lattice is such that $1 \wedge (4 \vee 5 \vee 6) = 1$ is "true" and then $4 \vee 5 \vee 6$ is "true," which for a Boolean-minded person means "either" 4 or 5 or 6 is "true." The paradox arises due to the absence of isomorphism of non-Boolean and Boolean structure. And here time plays a crucial role for the Boolean person. If there is no time objectively, the Boolean observer must "invent" it and "move" in it. He (she) can resolve the paradox by saying: at one moment t_1 I have 1 "true," 2–6 "false," at the other moment t_2 if I "measure" $4 \vee 5 \vee 6$ I will see some of 4, 5, 6 "becoming true." Then, for example, 4 "becomes" true; at this moment 1 "becomes" false because for Boolean-mindedness 4 "true" leads to 1, 2, 3, 5, 6 are false.

So truth values "jump" for a Boolean observer contemplating different Boolean substructures of a nondistributive lattice. This "jump" corresponds to the collapse of the wave function in measurement according to von Neumann. It is due to the discrepancy between non-Boolean and Boolean logics that "jumps" in truth values occur. This discrepancy also leads to "indeterminism" of quantum mechanics. If 1 is "true" at t_1, then 2–6 are "false" and so everything is determined. But because $4 \vee 5 \vee 6$ is "true," then at t_2 some totally undefined 4 or 5 or 6 "becomes" true for a Boolean mind. The Boolean mind cannot "predict" what element becomes true. And when he (she) "looks" at 4 or 5 or 6 he (she) will see according to Boolean logic the realization of some of the elements as true. Objectively existing potentiality then becomes "true" reality.

Here we come to the famous Wigner (1961) friend paradox. If truth values for atoms do not exist "objectively" and arise due to Boolean consciousness, one can ask: what about two Boolean-minded observers? It can be that he after "1 true at t_1" will say "4 true at t_2, 5, 6 false," but she will say "5 true, 4, 6 false." They both resolve the paradox for themselves in their way. Why do we as observers see the same quantum world? One of the answers can be that there is only one Boolean consciousness in which as observers all participate, or in Leibnitz' term there is one universal monad (God, according to Leibnitz') due to which all other monads can communicate and see one world, but not many.

The idea that consciousness plays an important role in quantum mechanics was expressed by many "fathers" of quantum theory. Von Neumann (1955) spoke about the Ultimate Observer, London and Bauer (1939) spoke about "introspection" leading to the collapse of the wave function, and Wigner (1961) also stressed the special role of consciousness in measurement. In order to make our point more clear, we must say that we do not think that there is "consciousness" in electrons, atoms, apparatuses, etc. When one asks: does a physical apparatus show something even if nobody looks at it? our answer will be: it shows what it shows, but "something" "shows," at this or that "moment of time"—all these words have "meaning" only for some Boolean mind. It is the same as with "greenness": trees are "green" for the human eye and only in this sense they were "green" millions of years before the origin of man.

This role of consciousness due to the difference of the logic of mind and the "logic" of the universe is important for such a property of time as "becoming" and "movement in time." Here we agree with Grünbaum (1973), who stresses just the same idea: "becoming" occurs due to the special role of consciousness. Nevertheless it is impossible to see any difference in the quantum properties of "nonconscious" (nonliving) matter and "conscious" (living) matter. The laws of physics are the same. But these "laws" are such for Boolean consciousness giving truth values to them.

4. ROLE OF TIME IN EPR EXPERIMENT

Let us investigate a two-particle system of two electrons with spin 1/2 for which only the $S_z^{(1)}$, $S_z^{(2)}$ components are measured. Here we try to give the answer to the paradox on Fig. 1 when one must say that $S_z^{(2)} = -1/2$ is not an "event" before another observer checked it. Draw the Hasse diagram for two particles as in Fig. 3. This non-Boolean lattice of questions corresponds to the following vectors in $\mathscr{H}_1 \otimes \mathscr{H}_2$:

$$\langle 1| = l_1 \otimes l_1; \qquad \langle 2| = l_1 \otimes l_2$$

$$\langle 3| = \frac{1}{\sqrt{2}}(l_1 \otimes l_2 - l_2 \otimes l_1)$$

$$\langle 4| = l_2 \otimes l_1; \qquad \langle 5| = l_2 \otimes l_2$$

A singlet state of the two-particle system is described by a collection of weights

$$w_1 = w_5 = 0, \qquad w_2 = w_4 = \frac{1}{2}, \qquad w_3 = 1$$

This lattice is nondistributive because

$$2 = 2 \wedge (4 \vee 3) \neq (2 \wedge 4) \vee (2 \wedge 3) = 0 \vee 0 = 0$$

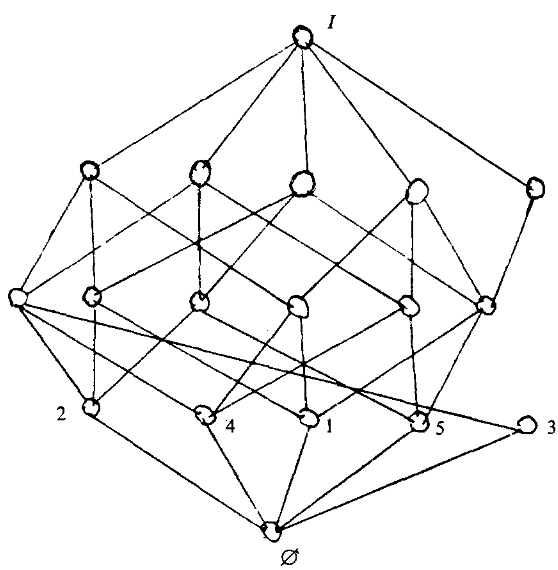

Fig. 3

When at some moment t_1 the system is prepared in $\langle 3|$, it is nevertheless impossible to call our weights "probabilities" because it is only after some observer "chooses" to measure at the "other moment" not $\langle 3|$ but $\langle 2|$ that probability for the Boolean sublattice arises. So the property $1 \vee 2$ corresponds to observation of $S_z^{(1)} = 1/2$ without observation of anything for the second particle. The occurrence of $S_z^{(1)} = 1/2$ does not mean that 2 occurs, because w_i still are not probabilities. It is only when another observer will "choose" to measure $S_z^{(2)}$ but not the permutation operator noncommuting with it (the eigenfunction of which is $\langle 3|$) that an "event" arises. So for the other observer it is also necessary to have two different times: the time of preparation t_0 and some other time $t_2 > t_0$ when he (she) chooses to measure $S_z^{(2)}$. Without this even if observer 1 says with "probability 1" that $S_z^{(1)} = 1/2$ it is only a "potentiality." So even if $t_2 > t_0$ it is necessary that at this t_2 "not permutation operator" is measured by the second observer.

Time has two meanings: the time of evolution described by the Schrödinger equation and "time of choice" due to an observer. The author is indebted to I. Prigogine, who in private communication stressed this difference of "two times."

In classical physics there is not this difference. But in classical relativistic physics there is no "becoming," everything "is" in space-time.

In quantum physics this difference is expressed by two different rules of using time: due to the Schrödinger equation and the wave function collapse. This difference can be even stronger in quantum cosmology, where sometimes people speak about the possibility of a time operator $\hat{T}$ noncommuting with the super-Hamiltonian $\hat{H}$. The Wheeler–DeWitt equation for quantum cosmology says $\hat{H}\Psi = 0$ and there is no "time of evolution." But it can be created by some "choice" to measure $\hat{T}$. Then one must invent a "fifth" dimension corresponding to a "time of choice" totally different from a "time of evolution." By these remarks we finish our investigation of the role of time in the quantum logical interpretation of quantum mechanics.

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